

A Space-Time Analytical Model for Energy Consumption in Wireless Sensor Networks

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Abstract—Sensor network *lifetime* suffers from increased energy consumption especially for those nodes in close proximity to the *sink node* that undertake rather the role of forwarding other nodes' data packets than their own. In this paper, a space-time analytical model is proposed to characterize the energy consumed in a sensor network under a rather simple medium access control policy. The proposed analytical model is studied and analyzed in this work revealing a *phase transition* phenomenon as the offered traffic load increases. Simulation results presented here are in compliance with the aforementioned analytical results.

I. INTRODUCTION

Sensor networks are composed of a number of small and comparably cheap devices, capable of sensing, computing and communicating, scattered in large numbers in certain areas of interest, [1]. There are numerous applications for sensor networks: fire detectors (e.g., inside buildings or in forest areas), nutrition level monitoring in agricultural fields, etc. Sensor networks are considered to be stationary (even though this is not mandatory), at least after their *deployment*. The deployment of sensors in an area of interest can be fixed when each node takes a predefined position (e.g., a fire detector in a building) or it can random when there is no predefined nodes positions.

After the deployment of a sensor network, nodes start sensing their surrounding environment. In case of a particular *event* of interest, the corresponding sensed information is conveyed to the *sink node* (the collector of information in sensor networks) over a *multihop* communication path composed of intermediate sensor nodes (i.e., nodes between the source node and sink node).

It is assumed that sensor nodes use *omni-directional* antennas and not any other sophisticated modern technology (e.g., antenna arrays, MIMO), due to the necessity

for the devices to remain small and cheap. Consequently, when a node transmits towards a neighbor node, the particular transmission may *interfere* with the transmissions of other nodes in the area resulting in a number of *corrupted* transmissions. If the rate according which data are sensed/generated in the network (i.e., the *load* of the network) is small, then data are expected to arrive at their final destination (i.e., the sink node) with a small probability to interfere with neighbor nodes. As the load of the network increases, some transmissions from neighbor nodes may take place simultaneously resulting in *corrupted* transmissions.

The avoidance of corrupted transmissions and increment of successful ones per time unit, is the main objective of most Medium Access Control (MAC) protocols proposed in the area of wireless networks (e.g., local area networks, ad hoc networks, sensor networks). In sensor networks, however, it is also important to suppress the number of corrupted transmissions due to certain *energy consumption limitations*. In particular, sensor nodes are usually equipped with a small battery that it may not be recharged at all (e.g., when nodes are scattered in a forest area). Therefore, it is important for the available energy to be used as efficiently as possible. Note that apart from transmission events, energy is also consumed for sensing, computing and receiving, [1], [2]. For the rest of this work, the focus is on the energy consumed during transmissions.

Several MAC protocols have been introduced in the area of wireless networks in an attempt to reduce the consumed energy. PAMAS, [4], refines mechanisms of the IEEE 802.11, [5], protocol to avoid interfering with other nodes in the network. Picoradio, [6], [7], allows data to be sent when the receiver is enabled, after a message over a special low power wakeup channel. S-MAC, [8], is a protocol addressing energy consumption issues in sensor networks. It allows for neighbor nodes to enter a sleep mode of operation when there is an ongoing transmission in the neighborhood and based on a simple scheduling algorithm it synchronizes the wakeup

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ans sleep modes of operation. STEM, [9], [10], is another protocol proposed in this area and utilizes a two-radio architecture for the efficient synchronization of the node wakeup and sleep modes of operation.

TDMA-based access schemes are also possible to be used to schedule transmissions in sensor networks. Under these schemes, nodes are assigned a certain time slot set for transmission purposes and be allowed to use the remaining time slots for entering the sleep mode. The problem is the particular assignment of the time slots to the node, [11], [12], even though topology-transparent MAC approaches could be used, [13], [14].

Even though there are many attempts for energy consumption minimization proposing a certain, for each case, MAC protocol, most of the researchers do not put any real attention on the *modelling*, of the transmission attempts (or, equivalently, the energy consumed during transmissions) in a sensor network, apart from [3] where an effort to analytically model the *hole* phenomenon (battery exhaustion for those nodes nearby the sink node) and [15] for a unitary disk. The aim of this paper is to proceed further by introducing a different and more generic analytical model that sheds light to certain aspects of the sensor network behavior. For example, it is a common belief that nodes near the sink node consume energy faster than other nodes due to the burden of relaying the information that of other nodes. Even though this is true in most of the cases, there are some exceptional cases that this is not valid, depending on the particular MAC protocol that is considered.

Our aim in this paper is to derive an analytical model for a sensor network, capable of capturing all different operational behaviors. Actually, as it is also shown, a sensor network is in a desired state of operation (i.e., most of the transmissions are succesful) for small values of the *traffic load*. However, there is a certain small range of values for the offered load (corresponding to a transient state of operation), above which transmission corruption is frequent resulting in energy consumption with no advantage since the *throughput* (i.e., number of successful transmission per slot for each node) drops dramatically.

The proposed analytical model follows new ideas and practices emerging in the literature (in many other fields i.e. materials science, [17]) which concerns multiscale dynamic modeling of complex systems. By the term complex systems we mean systems where their evolution in time is governed by the multiple (sort or long range) interactions between many single units (here among emitted nodes). As a result a variety of possible spatial

and temporal phenomena may arise, e.g., instabilities, critical phenomena, phase transformation and pattern formation.

The existence of sort or/and long range interactions between units drives the system in different behavior in different length scales. In a microscale, single behavior of any unit must be considered. While this kind of description is always accurate and take into account all possible interactions, in practice leads to unsolved problems due to the inherent randomness emerging in microscale. On the other hand, novel theories emerged in the literature, where deterministic system description proposed in a scale above, the mesoscale, by using appropriate averaging procedures of microscale randomness. As a result accurate knowledge of the underlying microsystem is sacrificed in order to capture main features of system behavior in macroscale, which most of the time is the desired outcome. Such phenomenon may be the *phase transformation* of network performance in space in relation of the emission mechanism (protocol) applying in microscale and load performance. To this end, we propose an appropriate deterministic evolution equation for the *information density per node* in mesoscale where the corresponding terms are extracted by considering explicit interactions taking place in the microscale. Even in this simplest form, we will demonstrate that the model is able to describe the *network performance* in time and space. In particular, for appropriate model parameters it is possible to predict localized or periodic solutions in macroscale as the value of network load increases leading thus to a phase transformation.

The employed MAC protocol is a rather simple one (actually all nodes are allowed to transmit during a time slot when there are data available for transmission). Furthermore, the considered network topology is actually a line, where the sink node is located at the one end. Generalization to more dimensions is straightforward. Both the aforementioned assumptions are made in order to simplify the analytical model and allow for its subsequent analysis. Simulation results are also presented in this paper and it was possible to observe the phase transformation phenomenon and identify the particular value for which the phenomenon takes place. As it is shown here, the proposed analytical model efficiently captures the system behavior revealed by the simulation results.

In Section II the system is described as well as the assumptions made in this paper. In Section III, the analytical model is presented and in Section IV it also analyzed. The simulation results are included in Section

V and the conclusions as drawn in Section VI.

II. SYSTEM AND ASSUMPTIONS

A sensor network may be viewed as a time varying multihop network and may be described in terms of a graph $G(V, E)$, where V denotes the set of nodes and E the set of (bidirectional) links between the nodes at a given time instance. Let $|X|$ denote the number of elements in set X and let $N = |V|$ denote the number of nodes in the network. Let S_u denote the set of neighbors of node u , $u \in V$. Let D denote the maximum number of neighbors for a node; clearly $|S_u| \leq D$, $\forall u \in V$. Set S_u includes any node v to which a direct transmission from node u (*transmission* $u \rightarrow v$) is possible. Let p be the *probability that there exist data available for transmission* during a time slot for any node in the network.

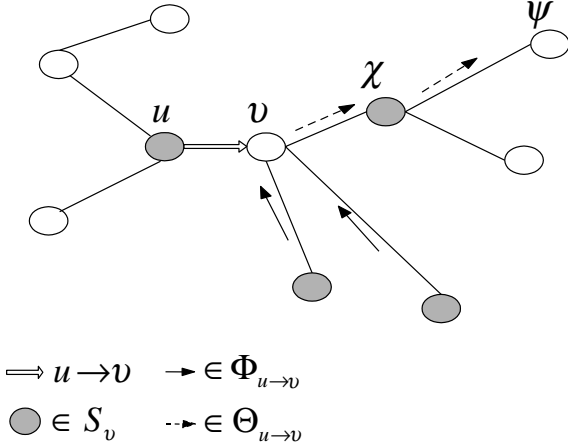


Fig. 1. Example transmission $u \rightarrow v$, set of nodes S_v and transmissions that belong in $\Phi_{u \rightarrow v}$ or $\Theta_{u \rightarrow v}$.

Suppose that node u wants to transmit to node v during a particular time slot i . Transmission $u \rightarrow v$ may be corrupted by any node that belongs to S_v (apart from node u). However, transmissions that corrupt transmission $u \rightarrow v$ may (set $\Phi_{u \rightarrow v}$) or may not (set $\Theta_{u \rightarrow v}$) be corrupted by it, as it is graphically depicted in Figure 1.

$$\Phi_{u \rightarrow v} = \left\{ \chi \rightarrow \psi : \chi \in S_v \cup \{v\} - \{u\}, \right. \\ \left. \psi \in S_\chi \cap (S_u \cup \{u\}) \right\}, \quad (1)$$

$$\Theta_{u \rightarrow v} = \left\{ \chi \rightarrow \psi : \chi \in S_v \cup \{v\} - \{u\}, \right. \\ \left. \psi \in S_\chi - (S_\chi \cap (S_u \cup \{u\})) \right\}. \quad (2)$$

It is common in sensor networks to assume that nodes are uniformly distributed in the area of interest. Consequently, $|S_v|$ is almost the same for all nodes in the network. This is a rather useful assumption since it allows for a more tractable form of the problem. Even though the aforementioned uniformity may not always be the case, it has been shown (e.g., in [14]), that the derived results may be used for non-uniform network, when certain conditions are satisfied.

During a certain transmission, nodes consume energy. If there is a corruption, it is evident that the consumed energy was not useful since the transmission attempt has to be repeated in a future time instance. If the corrupted transmission caused other transmissions to be corrupted (e.g., transmission $u \rightarrow v$ is corrupted and corrupts any other transmission $\chi \rightarrow \psi \in \Phi_{u \rightarrow v}$).

The MAC policy that is considered for the next of this paper is the following simple one.

Simple Policy: A node transmits during a time slot as long as there exist data available for transmission.

In the sequel, an analytical model is proposed considering for simplicity a line topology.

III. THE PROPOSED ANALYTICAL MODEL

In order to study the life time and energy consumption of an arbitrary sensor network we introduce as an appropriate state variable (without loss of generality we study 1-D sensor network when isotropy of the network is assumed) the *information density* (number of information packets per node) of a node n that is located at distance x from the sink node at time t .

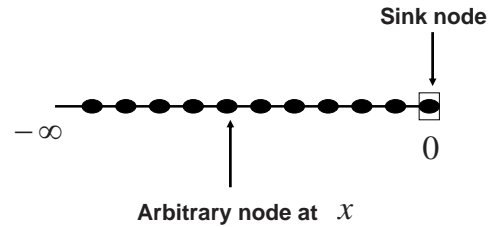


Fig. 2. In the 1-D network an arbitrary node $n(x, t)$ is located at distance x from the sink node.

According to the 1-D sensor network protocol the following evolution equation for the density of information $n(x, t)$ of an arbitrary node at location x and time t holds (assuming continuity of the network),

$$\dot{n}(x, t) = p + \alpha \frac{1}{n(x, t)} n(x-r, t) \frac{1}{n(x+r, t)} \\ - bn(x, t) \frac{1}{n(x+r, t)} \frac{1}{n(x+2r, t)}, \quad (3)$$

where $r = |x_i - x_{i-1}|$ is the spatial distance between two nodes and α, b rate constants which may depend on sensor network parameters (like p for example). The first term on the right hand side of Equation (3) stands for the constant flow of information generated at the particular node. As it was mentioned before in Section II, each node transmits information, generated either by itself or by other nodes, that needs to be relayed.

The second term of Equation (3) models the increase of information density (i.e., the *amount* of information due to transmissions towards the sink node) at each node due to successful receptions from its immediate left neighbor node, as it is the case depicted in Figure 2. This is (a) inversely proportional to the information density at a node at distance x from the sink node (the smaller the information density, the smaller probability of transmission and as a result the higher the probability of success); (b) analogous to the information density at a node at distance $x-r$ (the higher the information density, the higher the probability of a successful transmission); (c) inversely proportional to information density at a node at distance $x+r$ (the higher the information density, the smaller the probability of a successful transmission – collisions may take place due to the increased number of simultaneous transmissions). Finally, the third term of Equation (3) models the reduction of the information density due to successful transmissions towards the node on the right (see Figure 2). This is (a) analogous to the information density at a node at distance x (the higher the information density, the higher probability of transmission); (b) inversely proportional to the information density at a node at distance $x+r$; (c) inversely proportional to the information density at a node at distance $x+2r$ (the higher the information density, the smaller the probability of successful transmissions since collisions may take place).

It should be noted that the model described above is a *first approximation* regarding network performance since the linear dependence between information density and the probability of a successful transmission deteriorates when the load is increased. As a result a *load dependence* of the model parameters a, b is expected. This point will be clarified in the next section.

Performing a Taylor expansion around x the appearing densities in Equation (3) have in a first approximation the following forms (for smooth spatial densities the first

spatial derivative is negligible):

$$\begin{aligned} n(x+r, t) &= n(x-r, t) = n(x, t) + \frac{1}{2}r^2 \frac{\partial^2 n}{\partial x^2}, \\ n(x+2r, t) &= n(x, t) + 2r^2 \frac{\partial^2 n}{\partial x^2}. \end{aligned} \quad (4)$$

Substituting in Equation (3) and using $\frac{1}{1+y} \approx 1-y$,

$$\dot{n} = p + (\alpha - b) \frac{1}{n} + c(n) \frac{\partial^2 n}{\partial x^2}, \quad (6)$$

where the abbreviation $n = n(x, t)$ was used and the gradient coefficient has the following form: $c(n) = \frac{5r^2 b}{2n^2}$.

Equation (6) is the analytical expression for the analytical model. It describes the time evolution of the information density at an *arbitrary volume element* of a wireless network. The dependence of this evolution to the parameters of the network are explicitly considered. This type of evolution equations is common to other fields (e.g., see [16] in materials science) and have been extensively studied. The main feature of these equations is the pattern formation *in space* as a result of spatial interactions between nodes. The gradient coefficient is a measure of this interaction: The higher the value the stronger interaction.

IV. ANALYSIS

To describe the stationary information density pattern we study the steady-state solutions of the (evolution) Equation (6). The steady state version ($\dot{n} = 0$) is,

$$c \frac{\partial^2 n}{\partial x^2} = p(n), \quad p(n) = (b-a)n - pn^2. \quad (7)$$

This belongs to a general class of equations, which has been studied in detail in [16]. Three types of stationary spatial solutions are possible: reversals, localized and periodic solutions as depicted in Figures 3, 4 and 5. In the present work we are mainly interested in localized, around the sink node, and periodic solutions.

Since a simple MAC policy was adapted there is a linear dependency between the percentage of transmission and the information density per node. Without loss of generality unity was chosen as the corresponding linear coefficient. For this scenario, localized solutions are found for model parameters, $p = 10^{-3}$ and $b-a = 7 \times 10^{-5}$. Far away from the sink node (i.e., the distance $\rightarrow \infty$), the percentage $n_\infty = 5 \times 10^{-4}$ is estimated. For the sink node the corresponding n_S is estimated to be equal to $n_S = 0.105$. For a second set of model parameters, $p = 3 \times 10^{-3}$ and $b-a = 6.1 \times 10^{-4}$, the corresponding percentages of transmission, $n_\infty = 3 \times 10^{-3}$ and $n_S = 0.305$ are estimated.

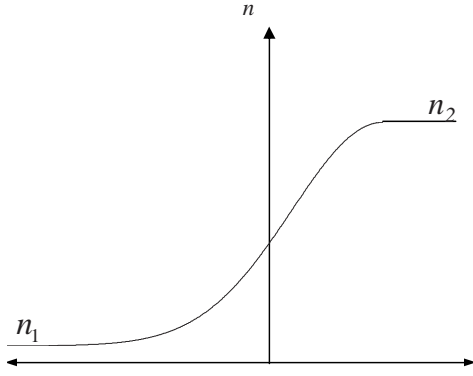


Fig. 3. Reversal type of solutions. The state variable n reverses its value in space, from n_1 to n_2 .

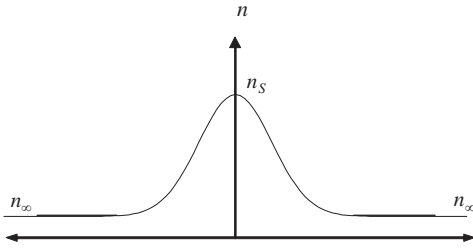


Fig. 4. Localized type of solutions. For low values of network load, the state variable n exhibits a localized maximum n_s around the sink node while approaches its limited value n_∞ at $\pm\infty$.

On the other hand, periodic solutions exist for all n that are in the range where the $p(n)$ plot exhibits a negative slope. When integrating Equation (6) twice gives,

$$x - x_0 = \int_{n(x_0)}^n \frac{1}{\sqrt{2F(u)}} du, \quad (8)$$

where

$$F(u) = \int_{n_1}^{n_2} p(u) \frac{1}{c(u)} du, \quad (9)$$

and x_0 is an arbitrary constant. For $n \in [n_1, n_2]$, Equation (8) describes a halfperiod of a periodic solution with wavelength,

$$q = 2 \int_{n_1}^{n_2} \frac{1}{\sqrt{2F(u)}} du. \quad (10)$$

For this scenario, periodic solutions are found for model parameters, $p = 10^{-3}$ and $b - a = 7 \times 10^{-5}$. Spatially, oscillation of the percentage of transmission between $n_1 = 0.0033$ and $n_2 = 0.00345$ is estimated.

As a result, the model predicts the emergence of two types of spatial patterns of the percentage of transmission in steady state: localized solutions with the maximum value located at the sink node and periodic solutions

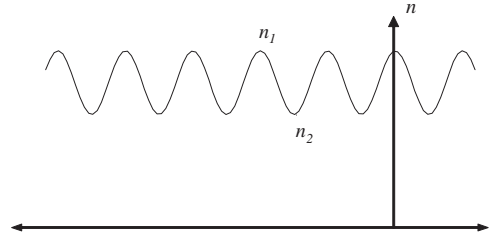


Fig. 5. Oscillation type of solutions. For greater values of network load, phase transformation takes place and periodic like solutions emerged, oscillating between successively local maximum and minimum state values, n_1 and n_2 correspondingly.

oscillating between a maximum and minimum value. Since the proposed formalism does not provide a definite value for the model parameters depending on specific probabilistic laws concerning transmissions and receptions between nodes, there is no clear way to predict which of the two possible patterns would be emerged. Intuitively, a kind of phase transformation is expected: for increasing network load p spatial patterns switch between localized and periodic like patterns.

While the proposed model in this initial form is not able to describe this expected transition, the values of the model parameters used by the proposed approach is found to follow a parabola,

$$c_1(p) = 10^{-4} - 0.13 \times p + 100 \times p^2, \quad c_1(p) = b - a. \quad (11)$$

As a result the final proposed partial differential equation for the stationary pattern of node transmissions is,

$$c(p) \frac{\partial^2 n}{\partial x^2} = c_1(p)n - pn^2. \quad (12)$$

V. SIMULATION RESULTS

For the simulation purposes a simulation program in C is created. The assumed topology considers 100 nodes in a line, the sink node being put at the end of the line for compliance with the assumptions made under the previously proposed analytical model. Each node is equipped with a large queue storing data packets waiting for transmission towards the sink node. Simulation experiments have been carried out for various values of the offered traffic load p and for 100 time slots time duration.

Figure 6 presents simulation results regarding the (average) transmission percentage during a time slot, as a function of the number of hops x that a node is located away from the sink node. For rather small values of p there is a decrement of the transmission percentage as x decreases (i.e., coming closer to the

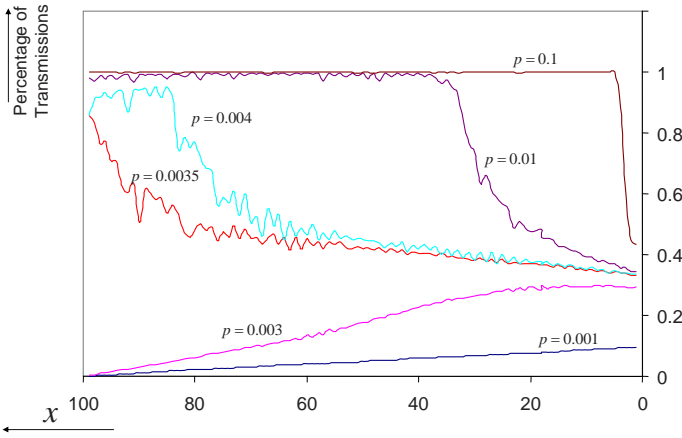


Fig. 6. Average number of transmissions (per time slot) as a function of the number of hops (away from the sink node) for different values of p .

sink node). This actually corresponds to the solution of the model depicted in Figure 4. As p increases the same shape of the corresponding curve is maintained. However, we observe a tendency to converge towards 0.4 for $x \rightarrow 1$ (i.e., close to the sink node). As p slightly increases (e.g., $p = 0.0035$), we observe a rather significant improvement of the overall transmission percentage for nodes located far away from the sink node. As x increases we still observe the aforementioned tendency to converge towards 0.4. As p increases further many more nodes located away from the sink node increase significantly their corresponded transmission percentage. This actually is partially captured by the solution of the model depicted in Figure 5. For example, for $p = 0.004$, those nodes located 80 or more hops away from the sink node transmit during almost every time slot. For even higher values of p (e.g., $p = 0.01$), nodes located 30 or more hops away from the sink node transmit during almost every time slot. For $p = 0.01$, this is the case for almost all nodes in the network (except a few close to the sink node). However, it is clear that the corresponding curves more or less converge towards 0.4, for $x \rightarrow 0$.

This particular behavior and the observed phase transformation phenomenon (i.e., that for some probability value p_C like $p_C = 0.0035$ in Figure 6, the system behavior radically changes), is in compliance with analytical results provided in the previous section and an indication that the proposed analytical model captures the main aspects of the system's behavior.

The transmission percentage, presented in Figure 6, is of significant importance in sensor network since it is closely related to the consumed energy. On the other hand, it is important to observe simulation results with

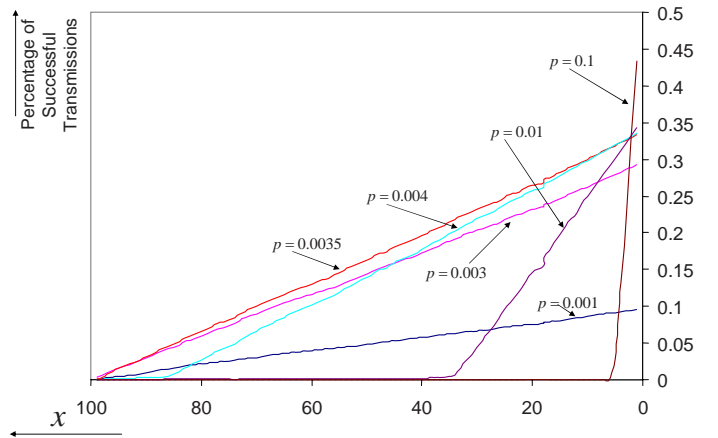


Fig. 7. Average number of transmissions (per time slot) as a function of the number of hops (away from the sink node) for different values of p .

respect to the (average) successful transmission rate or throughput. Therefore, in Figure 7 simulation results are presented regarding the percentage of successful transmissions of nodes as a function of the number of hops x . It is interesting to observe that for small values of p (e.g., $p = 0.001$), the throughput increases as x decreases. As p increases, the depicted throughput increases as well. This tendency to increase is maintained until $p = p_C$ (until the phase transformation phenomenon starts). For any offered load $p > p_C$, the corresponded throughput for those nodes far away from the sink node (e.g., $x > 80$ for $p = 0.004$) remains close to zero. As p increases further (e.g., $p = 0.01$), the number of nodes that their corresponded throughput is close to zero increases as well (e.g., $x > 40$ for $p = 0.01$).

When considering the results depicted in Figure 6 and Figure 7, it is easy to observe that for those nodes that the transmission percentage is close to one (in Figure 6). Clearly, this is the case where the corresponded nodes are not able to forward their stored data packets due to transmission collisions. Therefore, it is obvious that the system should operate under traffic load conditions smaller than the one corresponding to phase transformation.

VI. CONCLUSIONS

The comparison of the simulation results with the theoretical outcomes of the proposed model is in agreement with the discussion at the end of Section IV. Indeed, phase transitions for increasing network load was reported. For low values of the network load p , localized spatial patterns for the percentage of transmission in steady state were observed while for higher values of p ,

oscillation patterns emerge. For even higher values of p a new kind of spatial patterns emerge, leading the overall network performance to saturation. This behavior implies the existence of critical values for which the performance of the network is drastically changed. Specifically, for a critical value p_C of the network load, switch between localized and periodic like patterns take place. While the proposed model is able to describe this phase transformation of the spatial pattern, there is no definite way to estimate the critical value p_C . This is an important weakness of the proposed formalism and lies on the fact that the corresponding model coefficients were introduced in a more or less phenomenological way, i.e. there are not yet proposed specific probabilistic laws for the rates of emission / reception between neighbourhood nodes (e.g. dependence on the load of the network).

Note also that spatial pattern of the percentage of transmission as depicted from simulation results exhibit a departure from a deterministic behaviour. This is evidence mainly in the vicinity of the phase transformations and is a well known phenomenon (observed also in other fields) associated with critical random/stochastic interactions between nodes. This implies the need for the generalization of the proposed model (Equation (12)) by the introduction of appropriate stochastic terms.

Future work in this area will focus on the elaboration of the proposed model in order to introduce the corresponding model parameters with a more rigorous way than here. We expect that this can be done either with the introduction of specific probabilistic laws for the number of emissions / reception between neighbourhood nodes or by using tools developed under random graph theory.

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